Reply to Comments of Mandel and Wolf

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is homogeneous, i.e. it has no preferred origin of space. It does not therefore represent light emanating from a scattering centre, as Jakeman and Pike imply in their introduction.

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The objections raised by Mandel and Wolf to the use of the words 'quantum-mechanical' in our paper (Jakeman and Pike 1968, to be referred to as JP) follow a familiar pattern (see for example their criticism (Mandel and Wolf 1966) of Morawitz (1965)) and have been much discussed in the literature. Their arguments can be broadly divided into two classes: those relating to the choice of the measured quantity $E(T)$ and those concerned with the description of the field and the ensuing distribution-function calculation.

It is not possible to prove classically that the probability of emission of a photoelectron in time Δt, which is the quantity usually measured in photon-counting experiments, is proportional to the c number $\mathscr{E}^+(t)\mathscr{E}^-(t)$ (Einstein 1905). Arguments involving the classical envelope function (analytic signal) cannot be rigorous since the quantum theory of photodetection by annihilation (Glauber 1963) shows that the associated envelope operator $\{\hat{\mathscr{E}}^+(t)\hat{\mathscr{E}}^-(t)+\hat{\mathscr{E}}^-(t)\hat{\mathscr{E}}^+(t)\}/2$, whose use has been suggested for noise-current spectra at optical frequencies by Eckstein and Rostoker (1955), contains an incorrect additional zero-point energy. A discussion of this problem has been given by Butcher and Ogg (1965).

The nature of the distribution-function calculation is determined by the initial choice of $E(T)$ as opposed to the quantity $E_c(T)$ of classical noise theory and by the form of the density operator for the field. The fact that the c-number algebra involved is identical with that in the solution of a certain classical problem is immaterial; the calculation is physically quantum-mechanical, as has been pointed out by Klauder and Sudarshan (1968, page 192). We are therefore justified in labelling $E(T)$ and our calculation of its fluctuation distribution as quantum-mechanical.

The serious question here is whether in the general case the calculation can be performed, not classically, but semi-classically in the precise sense normally used (Schiff 1955, chap. 10); that is where everything is treated quantum-mechanically but the radiation field, which is treated classically. The answer to this question is now established to be a definite no. An arbitrary field must be dealt with in terms of its quantum-mechanical density matrix. If the problem is expressed in the so-called P representation the algebra is formally similar to a classical treatment of the field (though perhaps including negative and singular probabilities), but this in no way implies that such a calculation is not quantum-mechanical (Klauder and Sudarshan 1968).

In answer to the more specific point raised by Mandel and Wolf in regard to equation (6.13) of their review paper (1965) for the photon-counting distribution in the limit $\gamma\,(=T/\tau_c) \gg 1$, we remark that this is based on an approximate formula proposed by Rice (1945) in a section of his paper devoted to the evaluation of the probability distribution of $E_c(T)$. (Confusion should not arise because Rice compares the distribution $P(E_c)$ in the limit $\gamma \gg 1$ with the distribution of the envelope function in the opposite limit $\gamma \ll 1$.) This proposed formula of Rice for $P(E_c)$ leads to a photon-counting distribution which coincides with the correct quantum-mechanical expansion for $P(E)$ only to order $1/\gamma$ and in fact represents the exact probability distribution, for integral values of γ, of this number of statistically independent geometric distributions. For the case of a Lorentzian spectrum the analysis presented in JP can be used to obtain the correct quantum-mechanical result as an expansion in $1/\gamma$; this is

$$p(n,T) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}\left(1+\frac{1}{2\gamma}\{(n-\bar{n})^2-n\}+\frac{1}{8\gamma^2}[\{(n-\bar{n})^2-n\}^2-2(2\bar{n}+1)\{(n-\bar{n})^2-n\}\right.$$
$$\left. -2n(n-1)]+O\left(\frac{1}{\gamma^3}\right)\right).$$

The term in $1/\gamma$ has been given previously by McLean and Pike (1965) and should be used in preference to (6.13) of Mandel and Wolf (1965) since the significance of the interpretation of the above expansion in terms of distributions of bosons is illusory. It is the choice of density matrix and not the statistical properties of photons which gives rise to this formula. The second- and higher-order terms obtained by expanding the formula of Mandel and Wolf are incorrect.

Finally, in answer to the comment on equation (4) of JP we should like to point out that it is usual to analyse the scattering of light by a medium by considering (homogeneous) fluctuations of dielectric constant with a given wave vector (see, for example, Einstein 1910, Brillouin 1922, Landau and Placzek 1934). Comparison of experiment and theory based on equation (4) of JP for laser light scattered by a system of Brownian particles shows good agreement (Jakeman *et al.* 1968).

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